Pre-Calculus Chapter P Finals Questions Name KEY

1. Use the function $f(x) = x^3 - 12x^2 + 44x - 48$ to answer the following questions

a. Find one of the 3 x intercepts of the function

$x = 2, 4, 6$

b. Find the local maximum of the function

$x = 2.8452988$

$y = 3.0792014$

c. Use "intersect" to find the value of x when y = 20 (i.e. solve the equation $f(x) = x^3 - 12x^2 + 42x - 38 = 20$)

$x = 7.3681194$

$y = 20$

2. Write an equation for a circle centered at (5,-2) with radius 3. Then convert the equation into y = form

$(x-5)^2 + (y+2)^2 = 3^2$

$$\boxed{y = -2 \pm \sqrt{9 - (x-5)^2}}$$

3. Solve the equation $3x^2 + 5x - 12 = 0$ by any method. (factor, quadratic formula, graph)

$(3x - 4)(x + 3) = 0$

$x = -3$ $x = 4/3$

4. Solve the equation using "intersect": $|x + 1| = -\frac{1}{2}x + 3$ (find both solutions)

$x = -8$
$y = 7$ $(-8, 7)$

$x = 1.33\overline{3}$
$y = 2.33\overline{3}$ $(1.3\overline{3}, 2.3\overline{3})$

5. . Solve using the quadratic formula (imaginary solutions): $x^2 - 7x + 15 = 0$

$$x = \frac{7 \pm \sqrt{49 - 4(1)(15)}}{2} = \frac{7 \pm \sqrt{-11}}{2}$$

$$x = \frac{7 \pm i\sqrt{11}}{2}$$

Pre-Calculus Chapter 1 Final Review Name

1. Solve the equation. Find 1 real solution and 2 imaginary solutions: $2x^3 + 5x^2 + 8x = 0$

$x = 0$ ✓

$x(2x^2 + 5x + 8) = 0$

$x = \frac{-5 \pm \sqrt{25 - 4(2)(8)}}{2(2)} = \frac{-5 \pm \sqrt{25 - 64}}{4}$

$= \frac{-5 \pm \sqrt{-39}}{4} = \boxed{\frac{-5 \pm i\sqrt{39}}{4}}$

2. The table below represents a quadratic function:

X	Y	D_1	D_2
3	8.5		
4	12	3.5	+1
5	16.5	4.5	+1
6	22	5.5	+1
7	28.5	6.5	+1
8	36	7.5	+1
9	44.5	8.5	

a. To the right of the y column, show that the second differences are constant see

b. Using regressions, find an equation for the function

Stat → Edit

L_1	L_2
3	8.5
4	12
⋮	⋮

Stat → Calc → QUAD REG

$y = .5x^2 + 0x + 4$

c. Using your answer from part B, find x if y = 500

$x = 31.496031$

$y = 500$

ALSO -31.496031 because of symmetry

3. For each function below, find the domain. Write your answer in interval notation:

a. $f(x) = \sqrt{x-5}$ b. $g(x) = \frac{x+3}{x+4}$ c. $h(x) = \frac{\sqrt{x+1}}{x^2-9}$ d. $p(x) = \frac{x^3}{8}$ $\mathbb{R}$ or

$[5, \infty)$ $(-\infty, -4) \cup (-4, \infty)$ $[-1, 3) \cup (3, \infty)$ $(-\infty, \infty)$

4. Use the graphing calculator to find the range of the function $f(x) = -16x^2 + 125x + 30$ Write your answer in interval notation.

2nd, Trace, Max ...

$x = 3.9062505$

$y = 274.14062$

5. Use the graphing calculator to find the range of the function $f(x) = \frac{5}{x^2-4}$ Write your answer in interval notation.

$(-\infty, -1.25] \cup (0, \infty)$

5b. Verify algebraically that y = -1 is not in the range.

$-1 = \frac{5}{x^2-4}$

$(x^2-4)(-1) = 5$

$x^2-4 = -5$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

NOT REAL

6. For each function, identify any points of discontinuity. Then state whether the discontinuity is removable (hole) or infinite (asymptote). If the function has more than 1 discontinuity, name each one individually

a. $f(x) = \frac{x^2-7x+12}{x-3} = \frac{\cancel{(x-3)}(x-4)}{\cancel{(x-3)}}$

Hole at $x=3$

$\boxed{x \neq 3}$

b. $g(x) = \frac{x^2+x-30}{x-6} = \frac{(x+6)(x-5)}{(x-6)}$

Asymptote at $x=6$

$\boxed{x \neq 6}$

c. $h(x) = \frac{x}{x^2-4}$ $\frac{x}{(x+2)(x-2)}$

Asymptotes at

$x=2$

$x=-2$

$\boxed{x \neq \pm 2}$

7. Identify the intervals on which the function is increasing and decreasing. Write answer in interval notation:

a. $f(x) = -5x^2 + 32x + 8$

$x = \frac{-32}{2(-5)} = \frac{-32}{-10} = 3.2$

increasing: $(-\infty, 3.2)$

decreasing $(3.2, \infty)$

b. $g(x) = x^3 - 7x^2 + 8x + 5$

Increasing: $(-\infty, .666)$ $(3.999, \infty)$

decreasing: $(.666, 3.999)$

8. Let $f(x) = 2x^2 + 7$ and $g(x) = \sqrt{x-3}$

a. Write an equation for f(g(x)) and simplify

$f(g(x)) = 2(\sqrt{x-3})^2 + 7$

$= 2(x-3) + 7$

$= 2x - 6 + 7$

$= \boxed{2x+1}$

b. Find the domain of f(g(x)). Write your answer in interval notation

$x \geq 3$

$[3, \infty)$

c. Write an equation for g(f(x)) and simplify

$g(f(x)) = \sqrt{(2x^2+7)-3}$

$= \sqrt{2x^2+4}$

b. Find the domain of g(f(x)). Write your answer in interval notation

$(-\infty, \infty)$

9. For each question below, find f(x) and g(x) such that f(g(x)) = h(x) (decompose the function)

a. $h(x) = 3(x+2)^3 - 7(x+2)^2 + 2$

$f(x) = 3x^3 - 7x^2 + 2$

$g(x) = (x+2)$

b. $h(x) = Ln(x^2 - 9)$

$f(x) = Ln(x)$

$g(x) = x^2 - 9$

c. $h(x) = \tan(\sqrt{2x})$

$f(x) = \tan(x)$

$g(x) = \sqrt{2x}$

10. For each function below, write an equation for the inverse function:

a. $f(x) = 2x^2 - 7$

$y = 2x^2 - 7$

$x = 2y^2 - 7$

$\frac{x-7}{2} = y^2$

$\boxed{f^{-1}(x) = \sqrt{\frac{x-7}{2}}}$

b. $g(x) = \frac{x}{x+3}$

$y = \frac{x}{x+3}$

$x = \frac{y}{y+3}$

$x(y+3) = y$

$xy + 3x = y$

$3x = y - xy$

$3x = y(1-x)$

$y = \frac{3x}{1-x}$

$\boxed{g^{-1}(x) = \frac{3x}{1-x}}$

c. $h(x) = 25(1.2)^x$

$y = 25(1.2)^x$

$x = 25(1.2)^y$

$\frac{x}{25} = 1.2^y$

$\log_{1.2}\left(\frac{x}{25}\right) = y$

$h^{-1}(x) = \log_{1.2}\left(\frac{x}{25}\right)$

d. $p(x) = \cos(x^2)$

$y = \cos(x^2)$

$x = \cos(y^2)$

$\cos^{-1}(x) = y^2$

$y = \sqrt{\cos^{-1}(x)}$

$p^{-1}(x) = \sqrt{\cos^{-1}(x)}$

e. $r(x) = \sqrt{2x+1}$

$y = \sqrt{2x+1}$

$x = \sqrt{2y+1}$

$x^2 = 2y + 1$

$x^2 - 1 = 2y$

$y = \frac{x^2-1}{2}$

$\boxed{r^{-1}(x) = \frac{x^2-1}{2}}$

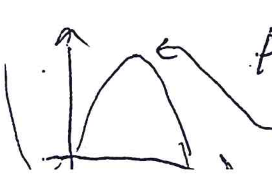
11. Transform the function $f(x) = x^2$ by dilating the function horizontally by a factor of 2, shifting left 3 units, and reflecting across the x axis. Write your new function in g(x) notation

(0) x^2 (1) $\left(\frac{x}{2}\right)^2$ (2) $\left(\frac{x+3}{2}\right)^2$ (3) $-\left(\frac{x+3}{2}\right)^2$

12. Transform the function $f(x) = \sqrt{x}$ by dilating vertically by a factor of 4, dilating horizontally by a factor of 1/3, and shifting up 5 units. Write your new function in g(x) notation

(0) $\sqrt{x}$ (1) $4\sqrt{x}$ (2) $4\sqrt{\left(\frac{x}{(1/3)}\right)}$ (3) $4\sqrt{\left(\frac{x}{(1/3)}\right)} + 5$

13. Find the maximum ~~volume~~ AREA of a rectangle inscribed in the parabola $y = 81 - x^2$



AREA $= 2x(81 - x^2)$

Max: $(5.196155\overline{7}, 561.1846)$